Analytic model of a Regge trajectory in the space-like and time-like regions[1]

R. Fiore[a], L.L. Jenkovszky[b], V. Magas[b,c], F. Paccanoni[d], A. Papa[a]

[a] *Dipartimento di Fisica, Università della Calabria,*
Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza
I-87036 Arcavacata di Rende, Cosenza, Italy

[b] *Bogolyubov Institute for Theoretical Physics,*
Academy of Sciences of Ukraine
252143 Kiev, Ukraine

[c] *Department of Physics, Bergen University,*
Allegaten 55, N-5007, Norway

[d] *Dipartimento di Fisica, Università di Padova,*
Istituto Nazionale di Fisica Nucleare, Sezione di Padova
via F. Marzolo 8, I-35131 Padova, Italy

Abstract

A model for a Regge trajectory compatible with the threshold behavior required by unitarity and asymptotics in agreement with Mandelstam analyticity is analyzed and confronted with the experimental data on the spectrum of the ρ trajectory as well as those on the $\pi^- p \to \pi^0 n$ charge-exchange reaction. The fitted trajectory deviates considerably from a linear one both in the space-like and time-like regions, matching nicely between the two.

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Regge trajectories may be considered as building blocks in the framework of the analytic S-matrix theory. We dedicate this contribution to the late N.N. Bogolyubov, whose contribution in this field is enormous, on the occasion of his 90-th anniversary. The model to be presented is an example of the realization of the ideas of the analytic S-matrix theory.

There is a renewed interest in the studies of the dynamics of the Regge trajectories [1, 2, 3]. There are various reasons for this phenomenon.

The hadronic string model (see e.g. [4]) was successful as a mechanical analogy, generating a spectrum similar to that of a linear trajectory, but it fails to incorporate the interaction between the strings. Although intuitively it seems clear that hadron production corresponds to breakdown of the strings, the theory of interacting strings faces many problems. Paradoxically, the final goal of the hadronic string theory and, in a sense, of the modern strong interaction theory, is the reconstruction of the dual (e.g. Veneziano) amplitude from the interacting strings, originated by the former.

Non-linear trajectories were derived also from potential models. The saturation of the spectrum of resonances was shown [5] to be connected to a screening quark-antiquark potential.

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A relatively new development is that connected with various quantum deformations, although the relation between q deformations and non-linear (logarithmic) trajectories was first derived by Baker and Coon [6]. q-deformations of the dual amplitudes (or harmonic oscillators) resulted [7, 8] in deviations from linear trajectories, although the results are rather ambiguous. By a different, so-called k-deformation, the authors [3] arrived at rather exotic hyperbolic trajectories.

All these developments were preceded by earlier studies of general properties of the trajectories [9], that culminated in classical papers of the early 70-ies by E. Predazzi and co-workers [10], followed by the paper of late A.A. Trushevsky [11], who were able to show, on quite general grounds, that the asymptotic rise of the Regge trajectories cannot exceed $|t|^{1/2}$. This result, later confirmed in the framework of dual amplitudes with Mandelstam analyticity [12], is of fundamental importance. Moreover, wide-angle scaling behavior of the dual amplitudes imposes an even stronger, logarithmic asymptotic upper bound on the trajectories. The combination of a rapid, nearly linear rise at small $|t|$ with the logarithmic asymptotics may be comprised in the following form of the trajectory:

$$\alpha(t) = \alpha(0) - \gamma \ln(1 - \beta t), \tag{1}$$

where γ and β are constants.

The threshold behavior of the trajectories is constrained by unitarity:

$$\mathcal{I}m\,\alpha_n(t) \sim (t - t_n)^{\mathcal{R}e\,\alpha(t_n)+1/2}, \tag{2}$$

where t_n is the mass of the $n-$th threshold. The combination of this threshold behavior with the square-root and/or logarithmic behavior is far from trivial, unless one assumes a simplified square root threshold behavior that, combined with the logarithmic asymptotics, results in the following form [13]

$$\alpha(t) = \alpha_0 - \gamma \ln(1 + \beta\sqrt{t - t_n}). \tag{3}$$

The next question is how do various thresholds enter the trajectory. In a long series of papers N.A. Kobylinsky with his co-workers [14] advocated the additivity idea

$$\alpha(t) = \alpha(0) + \sum_n \alpha_n(t), \tag{4}$$

with $\alpha_n(t)$ having only one threshold branch point on the physical sheet. The choice of the threshold masses is another controversial problem. Kobylinsky *et al.* [14] assumed that the thresholds are made only of the lowest-lying particles (and their antiparticles), appearing in the $SU(3)$ octet and decuplet - π and K mesons and baryons (N, eventually Σ and/or Ξ). We prefer to include the physical $4m_\pi$ threshold, an intermediate one at 1 GeV, as well as a heavy one accounting for the observed (nearly linear) spectrum of resonances on the ρ trajectory. The masses of the latter will be fitted to the data.

Fig. 1 shows the Chew-Frautschi plot with the trajectory (3), (4) and four thresholds [14] included. This trajectory matches well with the scattering data [16, 17, 18], as shown in Fig. 2, where fits to the scattering data based on the model [19] are presented.

The construction of a trajectory with a correct threshold behaviour and Mandelstam analyticity, or its reconstruction from a dispersion relation is a formidable challenge for the theory. This problem can be approached by starting from the following simple analytical model where the imaginary part of the trajectory is chosen as a sum of terms like

$$\mathcal{I}m\,\alpha_n(t) = \gamma_n \left(\frac{t - t_n}{t}\right)^{\mathcal{R}e\,\alpha(t_n)+1/2} \theta(t - t_n). \tag{5}$$

A rough estimate of $Re\,\alpha(t_n)$ can be obtained from a linear trajectory adapted to the experimental data. We have checked this approximation *a posteriori* and found that it works. It could be improved by iterating the zeroth order approximation. From the dispersion relation for the trajectory, the real part can be easily calculated [15]

$$\mathcal{R}e\,\alpha(t) = \alpha(0) + \frac{t}{\sqrt{\pi}} \sum_n \gamma_n \frac{\Gamma(\lambda_n + 3/2)}{\sqrt{t_n}\Gamma(\lambda_n + 2)}\, {}_2F_1\left(1, 1/2; \lambda_n + 2; \frac{t}{t_n}\right)\theta(t_n - t)+$$

$$+ \frac{2}{\sqrt{\pi}} \sum_n \gamma_n \frac{\Gamma(\lambda_n + 3/2)}{\Gamma(\lambda_n + 1)}\sqrt{t_n}\, {}_2F_1\left(-\lambda_n, 1; 3/2; \frac{t_n}{t}\right)\theta(t - t_n), \tag{6}$$

where $\lambda_n = \mathcal{R}e\,\alpha(t_n)$. Work in this direction is in progress.
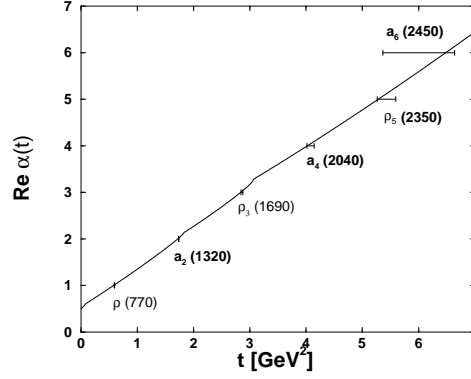
Figure 1: Chew-Frautschi plot for the six low-lying $I = 1$ parity even mesons (ρ-trajectory). The masses of the resonances were taken from [20].

References

[1] L. Burakovsky, *String model for analytic nonlinear Regge trajectories*, `hep-ph/9904322`; M.M. Brisudova, L. Burakovsky and T. Goldman, *Effective functional form of the Regge trajectories*, `hep-ph/9906293`.

[2] F. Filipponi, G. Pancheri, Y. Srivastava, Phys. Rev D **59**, 076003 (1999).

[3] J. Dey, P.L. Ferreira, L. Tomio and R.R. Choudhury, Phys. Lett. B **331**, 355 (1994); A. Delfino, J. Dey and M. Malheiro, Phys. Lett. B **348**, 417 (1995); J. Dey, M. Dey, P.L. Ferreira and L. Tomio, Phys. Lett. B **365**, 157 (1996).

[4] B.M. Barbashov and V.V. Nesterenko, *Relativistic String Model in Hadron Physics* (in Russian), (Energoatomizdat, Moscow, 1987).

[5] F. Paccanoni, S.S. Stepanov and R.S. Tutik, Mod. Phys. Lett. A **8**, 549 (1993); A.V. Kholodkov *et al.*, J. Phys. G **18**, 985 (1992).

[6] D.D. Coon, S. Yu and M. Baker, Phys. Rev. D **5**, 1429 (1972).

[7] M. Chaichian, J.F. Gomes and P. Kulish, Phys. Lett. B **311**, 93 (1993).

[8] L.L. Jenkovszky, M. Kibler and A.V. Mishchenko, Mod. Phys. Lett. A **10**, 51 (1995).

[9] V.N. Gribov and I.Ya. Pomeranchuk, Nucl. Phys. **38**, 516 (1962).

[10] A. Degasperis and E. Predazzi, Nuovo Cimento A **65**, 764 (1970); H. Fleming and E. Predazzi, Lett. Nuovo Cimento **4**, 556 (1970).

[11] A.A. Trushevsky, Ukr. Fiz. Zh. **22** 353 (1977).

[12] A.I. Bugrij et al., Fortschritte der Phys. **21**, 427 (1973).

[13] R. Fiore, L.L. Jenkovszky, V. Magas, F. Paccanoni, Phys. Rev. D **60**, 116003 (1999).

[14] See: N.A. Kobylinsky, V. Timikhin, Acta Physica Polonica B **9**, 149 (1977) and earlier references therein.

[15] *Tables of Integral Transforms* Vol. II, Ed. A. Erdelyi *et al.* (McGraw-Hill, New York, 1953)

[16] Serpukhov-CERN Collaboration (W.D. Apel *et al.*), Phys. Lett. B **72**, 132 (1977), JETP Lett. **26**, 502 (1977), Pisma Zh. Eksp. Teor. Fiz. **26**, 659 (1977).

[17] A.V. Barnes, D.J. Mellema, A.V. Tollestrup, R.L. Walker, O.I. Dahl, R.A. Johnson, R.W. Kenney, M. Pripstein, Phys. Rev. Lett **37**, 76 (1976)

[18] Serpukhov-Brussels-Annecy(LAPP) Collaboration (F. Binon *et al.*), Z. Phys. C **9**, 109 (1981)

[19] F. Arbab and C.B. Chiu, Phys. Rev. **147**, 1045 (1966).
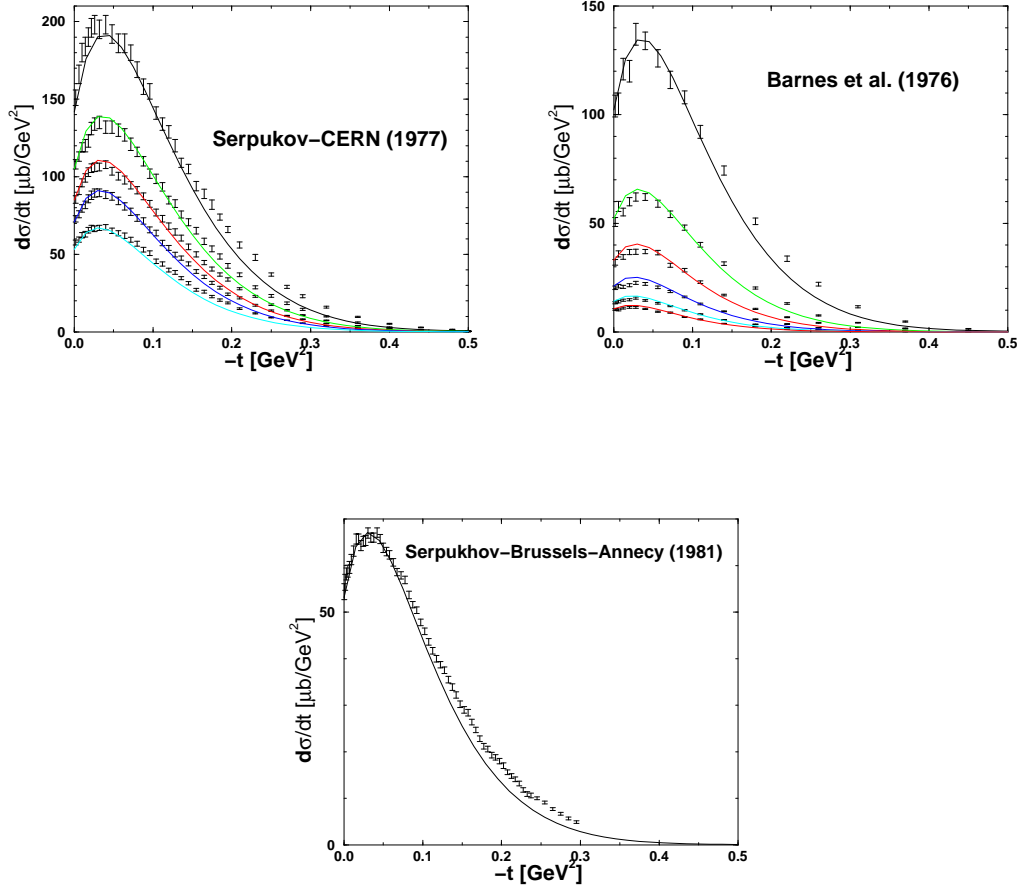
[20] C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

Figure 2: Differential cross section $d\sigma/dt$ [μb/GeV^2] versus $-t$ [GeV^2] for the process $\pi^- p \to \pi^0 n$. The solid curves represent the result of the fit with the model by Arbab and Chiu [19] using the trajectory defined in Eqs. (3) and (4). Data are taken from Ref. [16] (top-left), Ref. [17] (top-right) and Ref. [18] (bottom).